Exponents: the # of times a base is multiplied with itself — x^m (x = base, m = exponent)

Negative Eponents: the # of times 1 is divided by the base — $x^{-m} = \frac{1}{x^m}$

Fraction Exponents: radicals and exponents — $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$ — denominator is the index

Evaluate Without a calculator

2^3: $2\cdot2\cdot2 = 4\cdot2 = 8$

3^2: $3\cdot3 = 9$

4^{-1}: $\frac{1}{4}$

$2^{-4} = \frac{1}{16}$: $\frac{1}{2^4} = \frac{1}{2\cdot2\cdot2\cdot2}$

30^0: 1, $\frac{30^1}{30^1} = 1$

4^3: $4\cdot4\cdot4 = 16\cdot4 = 64$

$(-4)^3$ (odd): $-4\cdot-4\cdot-4 = 16\cdot-4 = -64$ negative

$(-4)^{-3}$: $\frac{1}{(-4)^3} = \frac{1}{-64}$

$(-2)^4$ (even): $-2\cdot-2\cdot-2\cdot-2 = 4\cdot4 = 16$ Positive

$(-2)^{-4} = \frac{1}{16}$: $\frac{1}{(-2)^4} = \frac{1}{16}$

$1^5 = 1$: $1\cdot1\cdot1\cdot1\cdot1 = 1\cdot1\cdot1$

$(-1)^5 = -1$: $-1\cdot-1\cdot-1\cdot-1\cdot-1 = 1\cdot1\cdot-1$

$(-1)^6 = 1$: $-1\cdot-1\cdot-1\cdot-1\cdot-1\cdot-1 = 1\cdot1\cdot1$

Exponential functions

Those in the form of

$$y = a\cdot b^{x-h} + k$$

, where b > 0 and x is a real number.

If the base was negative...., would it be a function?

$y = (-2)^x$

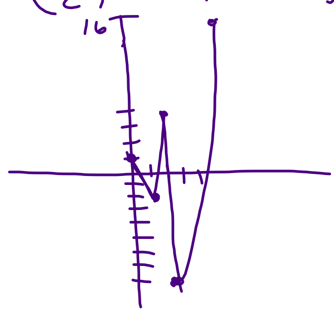
plug in 1: $(-2)^1 = -2$

plug in 2: $(-2)^2 = 4$ ($-2\cdot-2$)

in 3: $(-2)^3 = -8$ ($-2\cdot-2\cdot-2$)

$(-2)^4 = 16$

$(-2)^{1/2} = \sqrt[2]{-2}$ = imaginary ($+\cdot- = -2$)



Level 1

Evaluating Exponential Functions

Fractional a, Whole b value

$f(x) = 2(3)^x$ $a \cdot b^x$

$g(x) = 1/2(3)^x$ $a \cdot b^x$

use function f — f(-2)

$f(-2) = 2(3)^{-2}$
$f(-2) = 2 \cdot \frac{1}{3^2}$
$f(-2) = 2 \cdot \frac{1}{9}$
$f(-2) = \frac{2}{9} = y$
$(-2, \frac{2}{9})$

y int — f(0)

$f(0) = 2(3)^0$
$f(0) = 2(1)$
$f(0) = 2$
$(0, 2)$

f(2)

$f(2) = 2(3)^2$
$= 2(9)$
$f(2) = 18$
$(2, 18)$

use g function — g(-2)

$g(-2) = \frac{1}{2}(3)^{-2}$
$g(-2) = \frac{1}{2} \cdot \frac{1}{3^2}$
$g(-2) = \frac{1}{2} \cdot \frac{1}{9}$
$g(-2) = \frac{1}{18}$
$(-2, \frac{1}{18})$

y int — g(0)

$g(0) = \frac{1}{2}(3)^0$
$g(0) = \frac{1}{2}(1)$
$g(0) = \frac{1}{2}$
$(0, \frac{1}{2})$

g(2)

$g(2) = \frac{1}{2}(3)^2$
$= \frac{1}{2}(\frac{9}{1})$
$= \frac{9}{2}$
$g(2) = 4.5$
$(2, 4.5)$

'a' value affects y intercept.

$a \cdot 1 = a$

Level 2

Evaluating Exponential Functions

negative a or Fractional b value

$f(x) = -4(2)^x$

$g(x) = -4(1/2)^x$

f(-2)

$f(-2) = -4(2)^{(-2)}$
$= -4(0.25)$
$= -1$
$(-2, -1)$

f(0)

$f(0) = -4(2)^0$
$= -4(1)$
$= -4$
☆ $(0, -4)$

f(2)

$f(2) = -4(2)^2$
$= -4(4)$
$f(2) = -16$
$(2, -16)$

g(-2)

$g(-2) = -4(\frac{1}{2})^{-2}$
$= -4(\frac{2}{1})^2$ Flips!
$= -4(\frac{2^2}{1^2})$
$= -4(\frac{4}{1})$
$g(-2) = -16$
$(-2, -16)$

g(0)

$g(0) = -4(\frac{1}{2})^0$
$= -4(1)$
$= -4$
☆ $(0, -4)$

g(2)

$g(2) = -4(\frac{1}{2})^2$
$= -4(\frac{1^2}{2^2})$
$= -4(\frac{1}{4})$
$= \frac{-4}{4} = -1$
$(2, -1)$

Level 3: Model and Evaluate Exponential Functions

A factory purchased a 3D printer on January 2, 2010. The value of the printer is modeled by the function $f(x) = 30(0.93)^x$, where x is the number of years since 2010.

a. What is the value of the printer after 10 years?

$x = 10$

$f(10) = 30(0.93)^{10}$

$f(10) = 14.52$

b. Does the printer lose more of its value in the first 10 years or in the second?

$f(20) = 30(0.93)^{20}$

$f(20) = 7.03$

Value: $30 - 14.52$

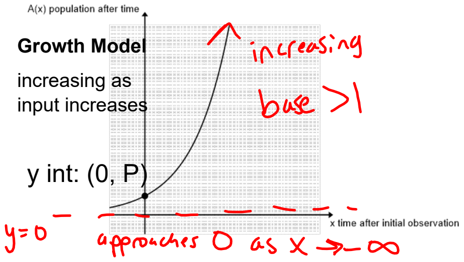
15.48

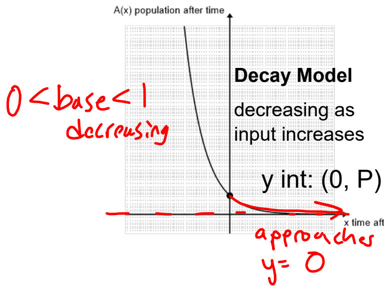
Value: $14.52 - 7.03$

7.49

it lost more value in the first 10 years

Exponential Growth/ Decay Equation	Function described by: $A = P(1 + r)^t$ A = Accumalated amount (ending amount) P: Principal amount (starting amount) r: rate of change as a decimal (positive if growth, negative if decay) % → decimal move point twice to the left t: time based upon rate in context





What is an Asymptote? an imaginary line that a function approaches but doesn't (usually) cross

Level 4 Interpret an Exponential Function

Two-hundered twenty hawks were released into a region in 2016. The function $f(x) = 220(1.05)^x$ can be used to model the number of red-tailed hawks in the region x years after 2016.

Note: How can you determine the growth or decay factor by looking at an exponential function? What about the growth or decay rate?

if base > 1 growth $r = b - 1$
if base < 1 decay $r = 1 - b$

4a. Is the population increasing or decreasing? Explain.

increasing

$1.05 > 1$

rate $= 1.05 - 1 = .05$

+5%

Positive rate

4b. In what year would you estimate the number of hawks to reach 280?

guess and check

$f(2) = 220(1.05)^2 = 242.5$

$f(5) = 220(1.05)^5 = 280.78$

5 or 4.95 years there will be 280 Hawks

During an economic recession, a charitable organization found that its donation dropped by 1.1% per year. Before the recession, its donations were $390,000.

a. Write an equation to represent the charity's donations since the beginning of the recession.

Step 1: A= ? P= 390,000 r= −0.011 t=

Step 2: Plug these in $A = P(1+r)^t$

$A = 390,000(1 + -0.011)^t$

$A = 390,000(0.989)^t$

less than 1 so decay

b. Estimate the amount of the donations 5 years after the start of the recession.

$t = 5$

$A = 390,000(0.989)^5$

$A = \$369,016.74$

$A(x) = P(1 + r)^x$

What if something has a 100% growth, what does that mean?

There are some famous exponential change models in mathematics

- When the rate of change is increasing by **100%, r = 1**
  - we have the **doubling model** : $A(x) = P(1 + 1)^x =$ **$P(2)^x$**
- When the rate of change is increasing by **200%, r = 2**
  - we have the **tripling model** : $A(x) = P(1 + 2)^x =$ **$P(3)^x$**
- When the rate of change is decreasing by **50%, r = - 0.5**
  - we have the **half model** : $A(x) = P(1 - 0.5)^x =$ **$P(0.5)^x$**

In Summary:

When the decimal version of the rate of change is **ADDED**,

we are building an exponential **GROWTH** model.

When the decimal version of the rate of change is **SUBTRACTED**,

we are building an exponential **DECAY** model.

Compound Interest

If the situation says: "compounded"...

$A = P(1+\frac{r}{n})^{nt}$

apron ant

A = Accumalated amount (ending amount)

P: Principal amount (starting amount)

r: rate of interest as a decimal (positive if growth, negative if decay)

n: number of times a year it is compounded

t: time

Compounded Meanings

the percent is taken from the newest amount and added to make a new total

compounded quarterly: n = 4

compounded monthly: n = 12

compounded weekly: n = 52

compounded annually: n = 1

compounded semiannually: n = 2

compounded biannually: n = 0.5

Determine the amount of an investment of $100,000 if it is invested at an interest rate of 5.2% compounded quarterly for 12 years.

Step 1: A= ? P= 100,000 r= 0.052 n= 4 t= 12

Step 2: Plug these in $A = P(1+\frac{r}{n})^{nt}$

$A = 100,000(1+\frac{0.052}{4})^{4 \cdot 12}$

$A = 100,000(1.013)^{48}$

A = $185,888.87

Determine the amount of an investment of $2500 if it is invested at an interest rate of 5.25% compounded monthly for 4 years?

P

r = 0.0525

n = 12

t = 4

$A = P(1+r)^t$ or $A = P(1+\frac{r}{n})^{nt}$?

$A = 2500(1+\frac{0.0525}{12})^{12 \cdot 4}$

$A = 2500(1.004375)^{48}$

A = $3,082.78