Header: Your Name

Mrs. Theo

3/3/22

Notes

# 8.1 - Ratios, Proportions, and Similarity

## 8.1 - Ratios, Proportions, and Similarity

**Ratio** Comparison between two numbers usually written as a quotient

| Fraction | Colon | Words |
|---|---|---|
| ~~1/2~~ $\frac{1}{2}$ | 1:2 | 1 to 2 |
| $\boxed{\frac{15}{5}}$ | $\boxed{15:5}$ | $\boxed{15 \text{ to } 5}$ |

**Proportion** Two equivalent ratios.

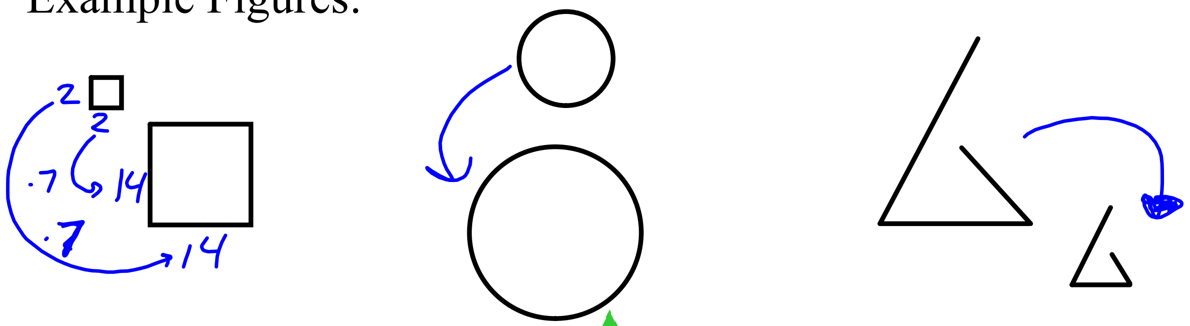
$$\frac{1}{2} = \boxed{\frac{7}{14}} \qquad \boxed{\frac{15 \div 5}{5 \div 5}} = \boxed{\frac{3}{1}}$$

$$\frac{15}{5} = \frac{9}{3} \quad \cdot 3$$

$$\frac{15}{5} \xrightarrow{\cdot 2} \frac{30}{10}$$

Similar Figures - Same shape, $\boxed{\text{Different}}$ size

Example Figures:



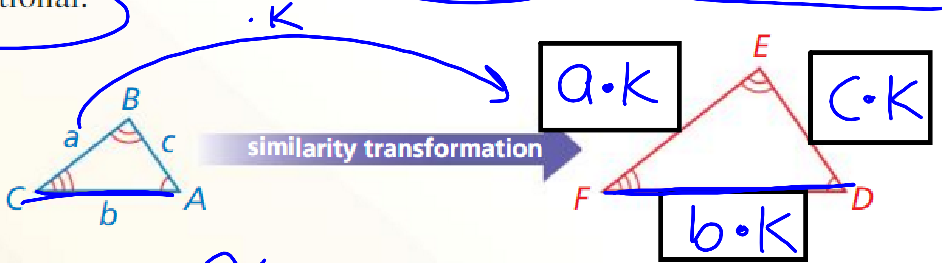

$\boxed{\text{Dilation}}$ = $\boxed{\text{Enlargement}}$
bigger

$\boxed{\text{Reduction}}$ = $\boxed{\text{Smaller}}$

# Core Concept

## Corresponding Parts of Similar Polygons

In the diagram below, $\triangle ABC$ is similar to $\triangle DEF$. You can write "$\triangle ABC$ is similar to $\triangle DEF$" as $\triangle ABC \sim \triangle DEF$. A similarity transformation preserves angle measure. So, corresponding angles are congruent. A similarity transformation also enlarges or reduces side lengths by a scale factor $k$. So, corresponding side lengths are proportional.




**Corresponding angles** ≅

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

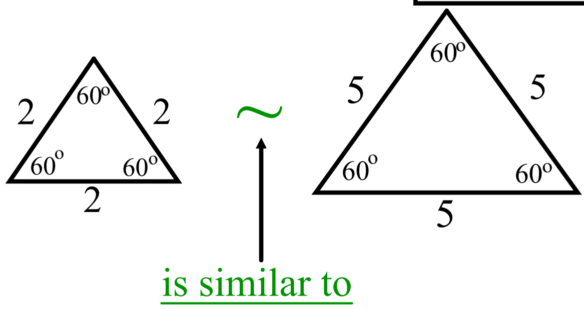
**Ratios of corresponding side lengths**

$$\frac{DE}{AB} = \frac{EF}{BC} = \boxed{\frac{FD}{CA}} = k$$

## Similar Polygons

1.) Corresponding angles are $\boxed{\text{congruent}}$

2.) The $\boxed{\text{ratios}}$ of the measures of corresponding sides are $\boxed{\text{equal}}$




## Scale Factor

K

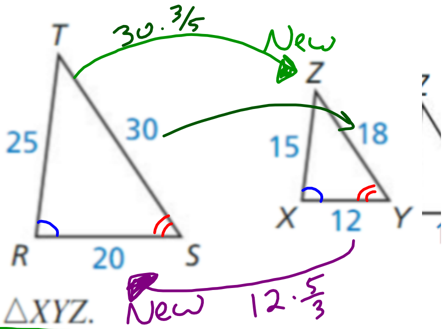
Factor (multiple) by which a figure is enlarged or reduced.

Divide: $\boxed{K = \frac{\text{New}}{\text{Old}}}$

Direction Matters

**Example #1** **Using Similarity Statements**

In the diagram, $\triangle RST \sim \triangle XYZ$.




**a.** List all pairs of congruent angles.

$\angle R \cong \angle X$

$\angle S \cong \angle Y$

$\angle T \cong \angle Z$

**b.** Find the scale factor from $\triangle RST$ to $\triangle XYZ$.

New

$K = \frac{\text{New}}{\text{old}} = \frac{18 \div 6}{30 \div 6}$

$\boxed{K = \frac{3}{5}}$ Reduction K is less than 1

**c.** Find the scale factor from $\triangle XYZ$ to $\triangle RST$.

New

$K = \frac{\text{New}}{\text{old}} = \frac{20}{12} = \boxed{\frac{5}{3}}$ Dilation

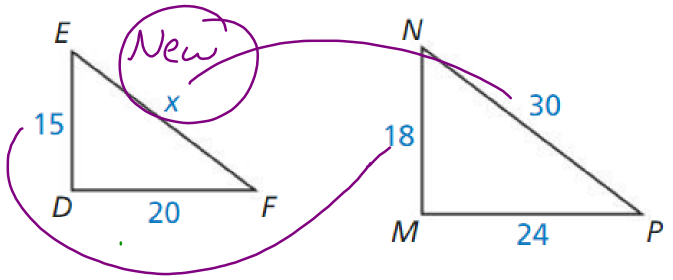
K is bigger than 1

$K = \frac{25}{15} = \frac{5}{3}$

$K = \frac{30}{18} = \frac{5}{3}$

## Example #2 Finding a Corresponding Length

In the diagram, $\triangle DEF \sim \triangle MNP$. Find the value of $x$.




Step 1: Determine Scale Factor $K = \frac{New}{old}$

$K = \frac{15}{18} = \boxed{\frac{5}{6}}$ ($\div 3$)

Step 2: Write a proportion
set $K = \frac{x}{\text{old side}}$

$\frac{5}{6} = \frac{x}{30}$ ($\cdot 5$)

$x = 25$
check for multiples

Step 3: Solve for x

Option 1
See if you can multiply by a number

Option 2
Cross multiply
multiply across = sign

$6 \cdot x = 5 \cdot 30$
$6x = 150$
$\frac{6x}{6} = \frac{150}{6}$
$x = 25$

$\frac{5}{6} \quad \frac{x}{30}$

$\frac{5x}{180}$

$\frac{5}{6} = \frac{x}{30}$

$6x = 150$

$\frac{30}{1}\left(\frac{5}{6}\right) = \left(\frac{x}{\cancel{30}}\right)\cancel{30}$

$\cancel{6}\left(\frac{30 \cdot 5}{\cancel{6}}\right) = (x)6$

$30 \cdot 5 = x \cdot 6$

$\frac{x+2}{x} = \frac{7x+3}{5}$

$5x + 10 = 7x^2 + 3x$

$$5\cdot\frac{x}{5}=\frac{20}{10}\cdot\frac{5}{1}$$

$$10\cdot(x)=\left(\frac{20\cdot 5}{\cancel{10}}\right)\cdot\cancel{10}$$

$$10\cdot x=20\cdot 5$$

$$\frac{10x}{10}=\frac{100}{10}$$

$$x=10$$

$$\frac{3}{5}=\frac{21}{35}$$

$$5\cdot 21=3\cdot 35$$

$$105=105$$

$$\frac{3}{5}\neq\frac{20}{35}$$

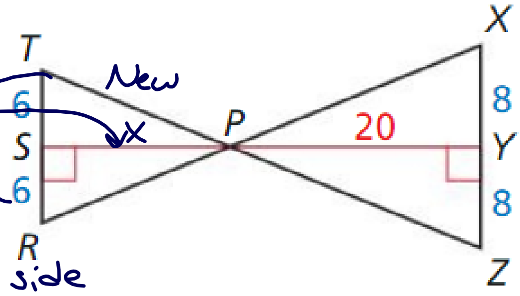
$$5\cdot 20=3\cdot 35$$

$$100\neq 105$$

Example #3 **Finding a Corresponding Length**

In the diagram, $\triangle TPR \sim \triangle XPZ$. Find the length of the altitude $\overline{PS}$.

segment from a vertex perpendicular to the opposite side



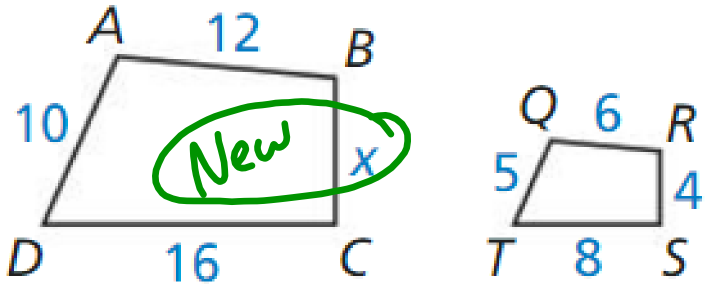

① $K=\frac{12}{16}\div 4=\frac{3}{4}$

② $\frac{3}{4}=\frac{x}{20}$ $\cdot 5$

$x=15$

# Example #4

Find the value of $x$.




$ABCD \sim QRST$

÷2

$$\frac{16}{8} = \frac{x}{4}$$

÷2

$x = 8$ ✓

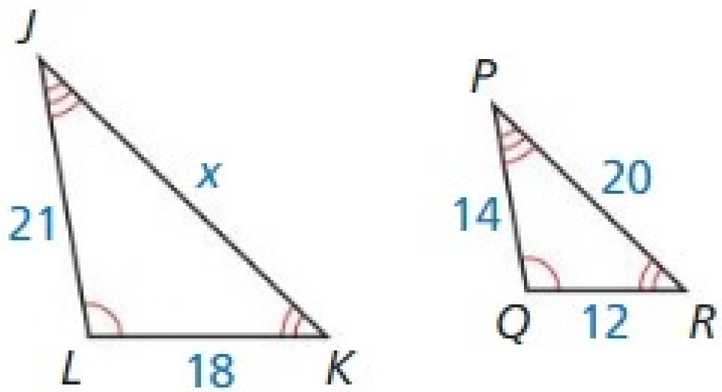
$$8x = 64$$

$$\frac{8x}{8} = \frac{64}{8}$$

$$\boxed{x = 8}$$

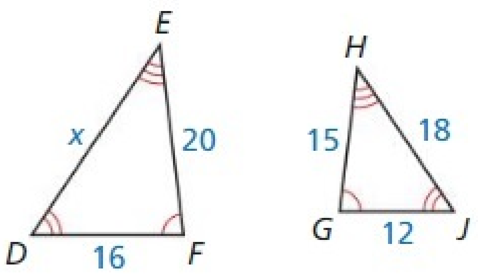
**In Exercises 5–8, the polygons are similar. Find the value of $x$.** *(See Example 2.)*

**5.**



**In Exercises 5–8, the polygons are similar. Find the value of *x*.** *(See Example 2.)*

**6.**

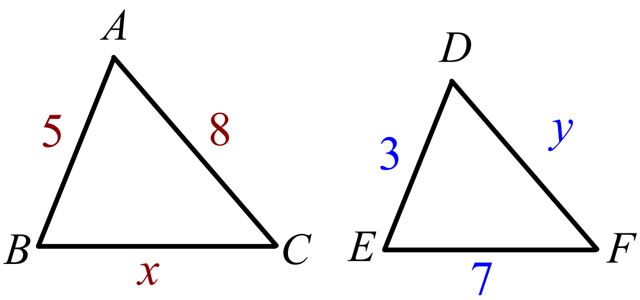



Warm UP

Find the values of $x$ and $y$.

$x = 11.\overline{6}, y = 4.8$

$\triangle ABC \sim \triangle DEF$




$\frac{y}{8} = \frac{3}{5}$

$5y = 24$

$y = 4.8$

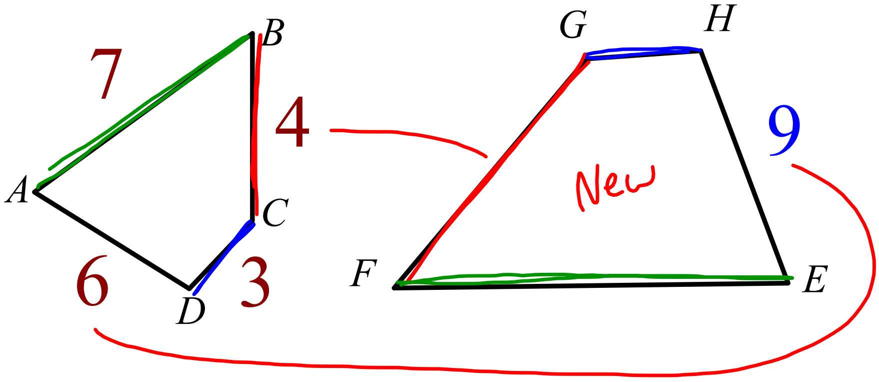
$\frac{x}{7} = \frac{5}{3}$

$3x = 35$

$x = 11.\overline{6}$

**Review** Find measures of the missing sides EF, FG, and GH.

ABCD ~ EFGH




$\frac{GF}{4} = \frac{9}{6}$

$6 \cdot GF = 36$

$\boxed{GF = 6}$

$\frac{FE}{7} = \frac{9}{6}$

$6 \cdot FE = 63$

$\boxed{FE = 10.5}$

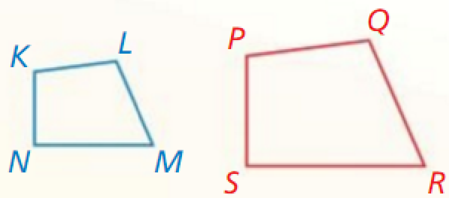
$\frac{GH}{3} = \frac{9}{6}$

$27 = 6 \cdot GH$

$\boxed{GH = 4.5}$

**Theorem Perimeters of Similar Polygons**

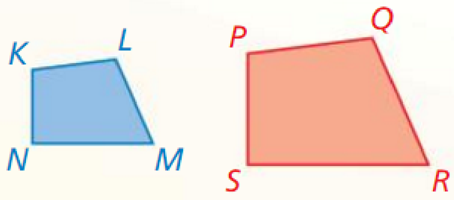
If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.




If $KLMN \sim PQRS$, then $\frac{PQ + QR + RS + SP}{KL + LM + MN + NK} = \frac{PQ}{KL} = \frac{QR}{LM} = \frac{RS}{MN} = \frac{SP}{NK}$.

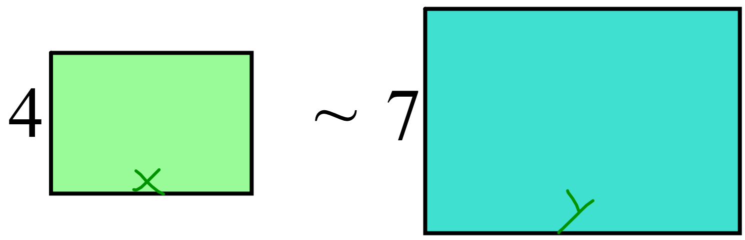
**Theorem Areas of Similar Polygons**

If two polygons are similar, then the ratio of their areas is equal to the squares of the ratios of their corresponding side lengths.




If $KLMN \sim PQRS$, then $\dfrac{\text{Area of } PQRS}{\text{Area of } KLMN} = \left(\dfrac{PQ}{KL}\right)^2 = \left(\dfrac{QR}{LM}\right)^2 = \left(\dfrac{RS}{MN}\right)^2 = \left(\dfrac{SP}{NK}\right)^2.$

## Other Ratios




Ratio of corresponding SIDES: $x : y$

$4:7 \qquad \frac{4}{7} = \frac{x}{y}$

Ratio of PERIMETERS: $x : y$

same as k ratio

$4:7 \qquad \frac{4+x+4+x}{7+y+7+y} = \frac{4}{7}$

Ratio of AREAS: $x^2 : y^2$

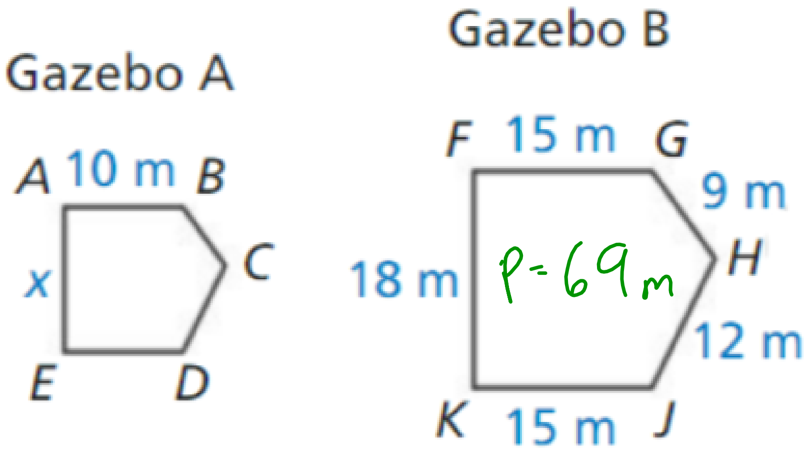
square each part of the ratio

$4^2 : 7^2 \qquad \frac{16}{49} = \frac{4x}{7y}$

$16 : 49$

## Example #7

The two gazebos shown are similar pentagons. Find the perimeter of Gazebo A.




$P = 15 + 9 + 12 + 15 + 18$

$K = \frac{10}{15}$

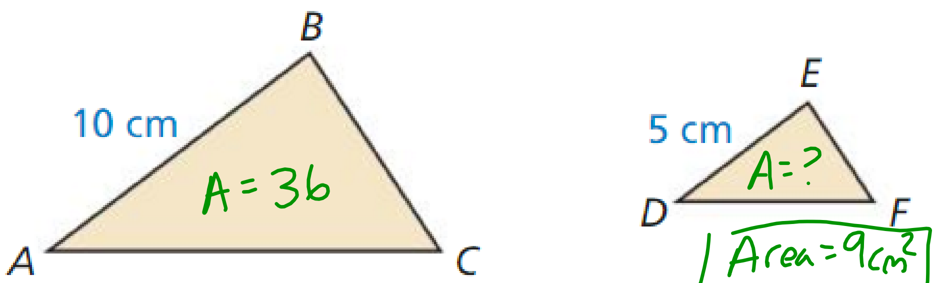
$\frac{10}{15} = \frac{P}{69}$

$15P = 690$

$\boxed{P = 46\text{ m}}$

## Example #8 Finding Areas of Similar Polygons

In the diagram, $\triangle ABC \sim \triangle DEF$. Find the area of $\triangle DEF$.




$\boxed{\text{Area} = 9\text{ cm}^2}$

Area of $\triangle ABC = 36\text{ cm}^2$

$K = \frac{5}{10} \xrightarrow{} \frac{5^2}{10^2}$

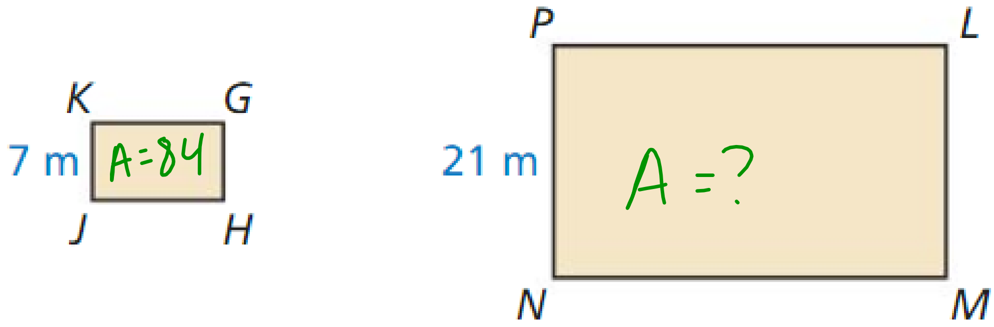
$\frac{25}{100} = \frac{A}{36}$

$100A = 900$

$A = 9$

## Example #9

In the diagram, $GHJK \sim LMNP$. Find the area of $LMNP$.




Area of $GHJK = 84\ m^2$

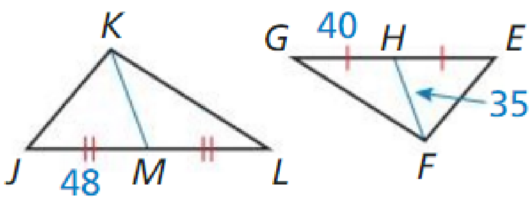
$K = \frac{21}{7} \rightarrow \frac{21^2}{7^2}$ $\frac{441}{49} = \frac{A}{84}$

$49A = 37044$

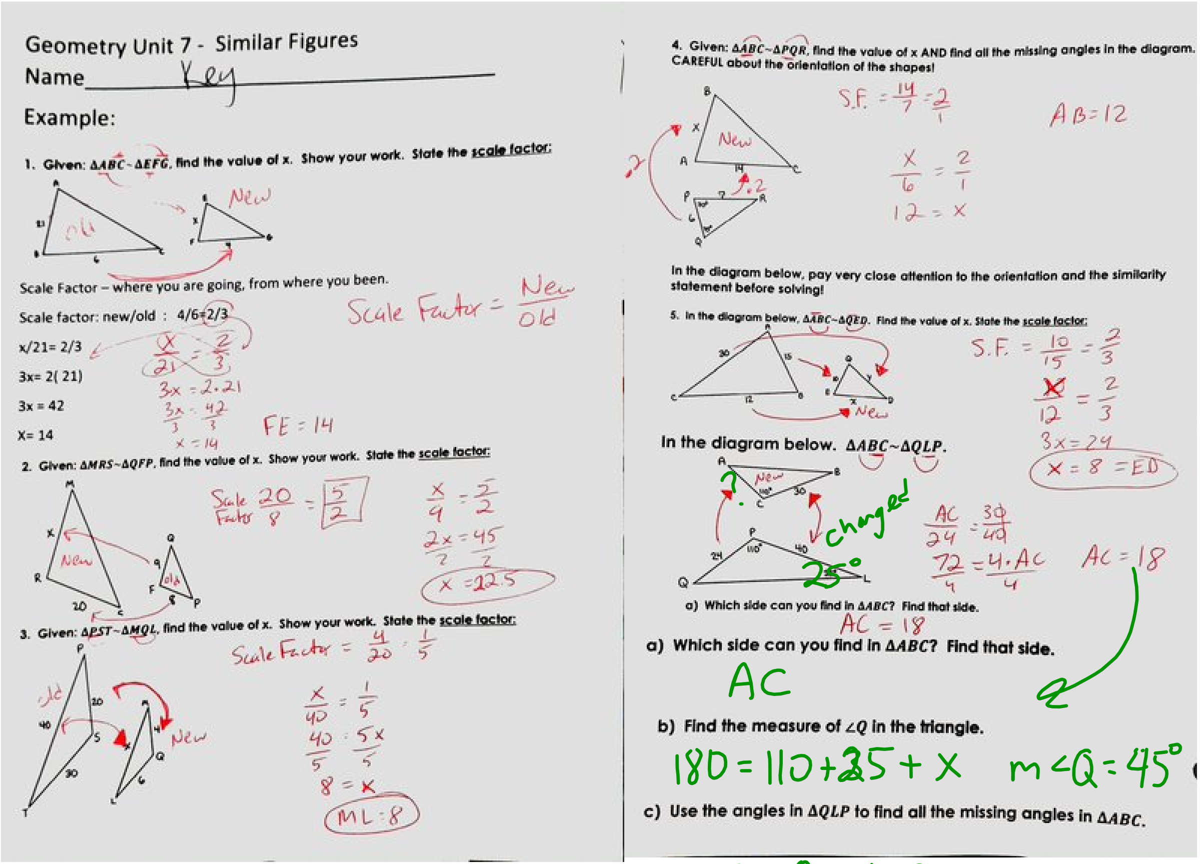
Area $= 756\ m^2$

## Example #7

Find $KM$.




$\triangle JKL \sim \triangle EFG$



Geometry Unit 7 - Similar Figures

Name Key

Example:

1. Given: $\Delta ABC \sim \Delta EFG$, find the value of x. Show your work. State the scale factor:

New old

Scale Factor – where you are going, from where you been.

Scale factor: new/old : 4/6=2/3

Scale Factor = New / Old

x/21= 2/3

3x= 2( 21)

3x = 42

X= 14

$\frac{x}{21} = \frac{2}{3}$

$3x = 2 \cdot 21$

$\frac{3x}{3} = \frac{42}{3}$

$x = 14$

FE = 14

2. Given: $\Delta MRS \sim \Delta QFP$, find the value of x. Show your work. State the scale factor:

Scale Factor $\frac{20}{8} = \boxed{\frac{5}{2}}$

$\frac{x}{9} = \frac{5}{2}$

$\frac{2x}{2} = \frac{45}{2}$

$x = 22.5$

3. Given: $\Delta PST \sim \Delta MQL$, find the value of x. Show your work. State the scale factor:

Scale Factor = $\frac{4}{20} = \frac{1}{5}$

$\frac{x}{40} = \frac{1}{5}$

$\frac{40}{5} = \frac{5x}{5}$

$8 = x$

ML = 8

4. Given: $\Delta ABC \sim \Delta PQR$, find the value of x AND find all the missing angles in the diagram. CAREFUL about the orientation of the shapes!

S.F. = $\frac{14}{7} = \frac{2}{1}$

AB = 12

$\frac{x}{6} = \frac{2}{1}$

$12 = x$

In the diagram below, pay very close attention to the orientation and the similarity statement before solving!

5. In the diagram below, $\Delta ABC \sim \Delta QED$. Find the value of x. State the scale factor:

S.F. = $\frac{10}{15} = \frac{2}{3}$

$\frac{x}{12} = \frac{2}{3}$

$3x = 24$

$x = 8 = ED$

In the diagram below. $\Delta ABC \sim \Delta QLP$.

changed 25°

$\frac{AC}{24} = \frac{30}{40}$

$\frac{72}{4} = \frac{4 \cdot AC}{4}$

AC = 18

a) Which side can you find in $\Delta ABC$? Find that side.

AC = 18

a) Which side can you find in $\Delta ABC$? Find that side.

AC

b) Find the measure of $\angle Q$ in the triangle.

$180 = 110 + 25 + x$ $m\angle Q = 45°$

c) Use the angles in $\Delta QLP$ to find all the missing angles in $\Delta ABC$.

$m\angle A = 45°$ $m\angle B = 25°$

Homework

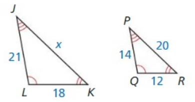
8.1 p.423

#5,6,17,18,28-30,35-38

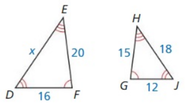
#11,12,16,23,24,31,32,34,51,55

**In Exercises 5–8, the polygons are similar. Find the value of $x$.** *(See Example 2.)*

**5.**

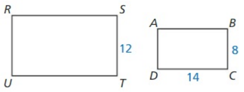



**6.**

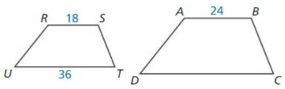



**In Exercises 11 and 12, $RSTU \sim ABCD$. Find the ratio of their perimeters.**

**11.**




**12.**



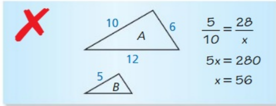

**In Exercises 13–16, two polygons are similar. The perimeter of one polygon and the ratio of the corresponding side lengths are given. Find the perimeter of the other polygon.**

**16.** perimeter of larger polygon: 85 m; ratio: $\frac{2}{5}$

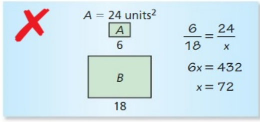
**17.** **MODELING WITH MATHEMATICS** A school gymnasium is being remodeled. The basketball court will be similar to an NCAA basketball court, which has a length of 94 feet and a width of 50 feet. The school plans to make the width of the new court 45 feet. Find the perimeters of an NCAA court and of the new court in the school. *(See Example 4.)*

**18.** **MODELING WITH MATHEMATICS** Your family has decided to put a rectangular patio in your backyard, similar to the shape of your backyard. Your backyard has a length of 45 feet and a width of 20 feet. The length of your new patio is 18 feet. Find the perimeters of your backyard and of the patio.

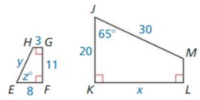
**23.** **ERROR ANALYSIS** Describe and correct the error in finding the perimeter of triangle B. The triangles are similar.




**24.** **ERROR ANALYSIS** Describe and correct the error in finding the area of rectangle B. The rectangles are similar.




**ANALYZING RELATIONSHIPS** **In Exercises 28–34, $JKLM \sim EFGH$.**




**28.** Find the scale factor of $JKLM$ to $EFGH$.

**29.** Find the scale factor of $EFGH$ to $JKLM$.

**30.** Find the values of $x$, $y$, and $z$.

**31.** Find the perimeter of each polygon.

**32.** Find the ratio of the perimeters of $JKLM$ to $EFGH$.

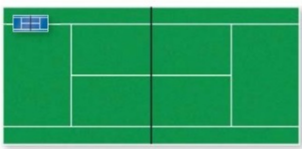
**33.** Find the area of each polygon.

**34.** Find the ratio of the areas of $JKLM$ to $EFGH$.

**35.** **USING STRUCTURE** Rectangle A is similar to rectangle B. Rectangle A has side lengths of 6 and 12. Rectangle B has a side length of 18. What are the possible values for the length of the other side of rectangle B? Select all that apply.

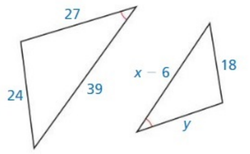
Ⓐ 6  Ⓑ 9  Ⓒ 24  Ⓓ 36

**36.** **DRAWING CONCLUSIONS** In table tennis, the table is a rectangle 9 feet long and 5 feet wide. A tennis court is a rectangle 78 feet long and 36 feet wide. Are the two surfaces similar? Explain. If so, find the scale factor of the tennis court to the table.

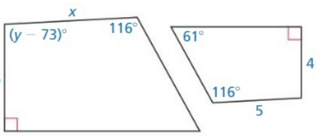


**MATHEMATICAL CONNECTIONS** **In Exercises 37 and 38, the two polygons are similar. Find the values of $x$ and $y$.**

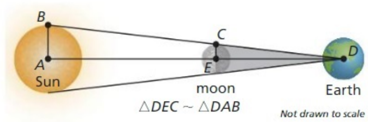
**37.**



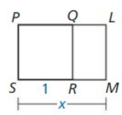

**38.**




**51.** **MODELING WITH MATHEMATICS** During a total eclipse of the Sun, the moon is directly in line with the Sun and blocks the Sun's rays. The distance $DA$ between Earth and the Sun is 93,000,000 miles, the distance $DE$ between Earth and the moon is 240,000 miles, and the radius $AB$ of the Sun is 432,500 miles. Use the diagram and the given measurements to estimate the radius $EC$ of the moon.



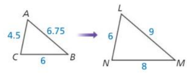

**55.** **CRITICAL THINKING** In the diagram, $PQRS$ is a square, and $PLMS \sim LMRQ$. Find the exact value of $x$. This value is called the *golden ratio*. Golden rectangles have their length and width in this ratio. Show that the similar rectangles in the diagram are golden rectangles.



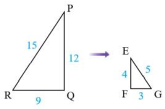
8.1 Similar Figures Homework pg. 423 Header:

**In Exercises 3 and 4, find the scale factor.**

3. $\triangle ABC \sim \triangle LMN$

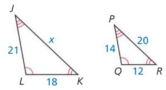



4. $\triangle PQR \sim \triangle EFG$

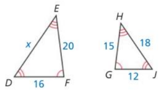



In Exercises 5 and 6, the given triangles are similar. Find the scale factor, $k$. Next, *use* $k$ to find the value of $x$. Finally, create a complete similarity statement between the two triangles.

5.




6.




**In Exercises 13–16, two triangles are similar. The perimeter of one triangle and the ratio of the corresponding side lengths are given. Find the perimeter of the other triangle.**

13. perimeter of smaller triangle: 48 cm; ratio: $\frac{2}{3}$

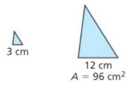
14. perimeter of smaller triangle: 66 ft; ratio: $\frac{3}{4}$

15. perimeter of larger triangle: 120 yd; ratio: $\frac{1}{6}$

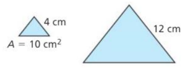
16. perimeter of larger triangle: 85 m; ratio: $\frac{2}{5}$

**In Exercises 19–20, the triangles are similar. The area of one triangle is given. Find the area of the other triangle.**

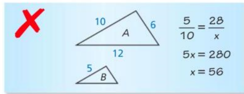
19.




20.




23. **ERROR ANALYSIS** Describe and correct the error in finding the perimeter of triangle B. The triangles are similar.




27. **REASONING** Triangles $ABC$ and $DEF$ are similar. Which statement is correct? Select all that apply.

Ⓐ $\frac{BC}{EF} = \frac{AC}{DF}$

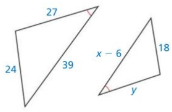
Ⓑ $\frac{AB}{DE} = \frac{CA}{FE}$

Ⓒ $\frac{AB}{EF} = \frac{BC}{DE}$

Ⓓ $\frac{CA}{FD} = \frac{BC}{EF}$

**MATHEMATICAL CONNECTIONS In Exercise 37, the two triangles are similar. Find the values of $x$ and $y$.**

37.



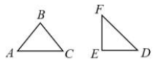

**(OPTIONAL)**

Prove that when two triangles are similar by a scale factor $k$, their perimenters are proportional by the same scale factor. Use the follow givens and diagram to start.

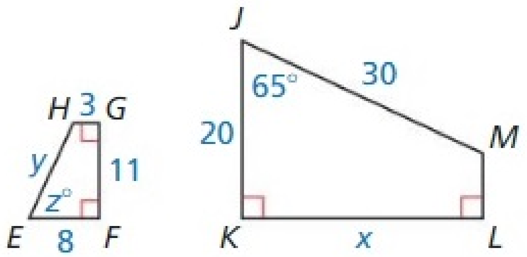
Given: $\triangle ABC \sim \triangle DEF$ by scale factor $k$.

$P_1 = a + b + c$ and $P_2 = d + e + f$

Prove: $P_1 = kP_2$




**ANALYZING RELATIONSHIPS** **In Exercises 28–34, *JKLM* ~ *EFGH*.**




28. Find the scale factor of $JKLM$ to $EFGH$.

29. Find the scale factor of $EFGH$ to $JKLM$.

30. Find the values of $x$, $y$, and $z$.