

Your Name

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Notes

Lesson 1.4 -
Solving Systems of
3 Variable Equations

Graphing, Checking,
and Substitution

Math Skill Objective:

- Visualize solutions of systems of equations in 3 variables
- Check if an ordered triple is a solution to the system.
- Solve systems of equations with 3 variables algebraically using substitution

Life Lesson: As you get older, your problems become bigger and more complicated. You must use what you know and extend that knowledge to solve the new and bigger problems you are faced with. One way is to put something aside so that you can solve a simpler problem you are familiar with first, and then go back and solve the entire problem.

A linear equation in three variables, x , y , and z is an equation of the form $ax + by + cz = d$ where a , b and c are not all zero.

A solution of such a system is an ordered triple (x, y, z) whose coordinates make each equation true.

The graph of a linear equation in three variables is a plane in 3-dimensional space.



$$0x + 0y + 0z = d$$

$$0 = d$$

3 Variable System

Types of Solutions

Ways Three Planes Can Intersect & Solution Implications
 (remember all three planes must intersect in order for it to be a solution)

3 Parallel Planes
NO solution

All 3 planes intersect INFINITE solutions (ALL points on line) at a line

2 parallel planes intersected by a plane
NO solution

3 planes intersect at different lines
NO solution

ONE solution, an ordered triple
All 3 planes intersect at a point

Which of these systems has a solution? How do you know?

① One Solution

② Infinite Solutions

③ No Solutions

④ No Solutions

⑤ Infinite Solutions

⑥ Infinite Solutions

⑦ No Solutions

⑧ No Solutions

One Solution

Infinite Solutions

No Solutions

Which of these systems has a solution? How do you know?

One Solution

Infinite Solutions

No Solutions

Checking if
an ordered
triple is a
solution

If an ordered triple works for all three equations when substituted in, then it is a solution to the system

Here is a **system of three linear equations** in three variables:

$$\begin{cases} x + 2y - 3z = -3 \\ 2x - 5y + 4z = 13 \\ 5x + 4y - z = 5 \end{cases}$$

The ordered triple $(2, -1, 1)$ is a **solution** to this system since it is a solution to all three equations.

$$\begin{cases} 2 + 2(-1) - 3(1) = 2 - 2 - 3 = -3 \quad \checkmark \\ 2(2) - 5(-1) + 4(1) = 4 + 5 + 4 = 13 \quad \checkmark \\ 5(2) + 4(-1) - 1 = 10 - 4 - 1 = 5 \quad \checkmark \end{cases}$$

Decide whether the given ordered triple is a solution of the system.

2. $(-1, -2, 5)$

$$2x + y - 5z = -29$$

$$6x + 4y - z = -19$$

$$x + y + 2z = 7$$

$(-1, -2, 5)$ is
a solution

$$2(-1) + (-2) - 5(5) = -29$$

$-2 \quad -2 \quad -25 \quad = -29 \checkmark$

$$6(-1) + 4(-2) - (5) = -19$$

$-6 \quad -8 \quad -5 \quad = -19 \checkmark$

$$(-1) + (-2) + 2(5) = 7$$

$-1 \quad -2 \quad 10 \quad = 7 \checkmark$

Solving 3 Variable System by Substitution

1. Pick the equation you will solve for one variable for and do so.
2. Substitute that expression in for the variable into the two other equations which will create two new equations with the same two variables
3. Solve the new system of two variables using elimination or substitution (again).
4. Plug the two solutions for the variables you found back in to one of the original 3 variable equations to solve for the third variable.

Ex 1: $x + 2y + 3z = 225$
 $x + y + z = 115$
 $y = 4x$

step 1
 $y = 4x$

step 2
 $x + 2(4x) + 3z = 225$
 $x + 8x + 3z = 225$
 $9x + 3z = 225$

$x + (4x) + z = 115$
 $5x + z = 115$

two new equations w/ same two variables. Choose your method: Substitution or Elimination?

step 3
 $-15x - 3z = -345$

$\frac{-6x}{-6} = \frac{-120}{-6}$
 $x = 20$

$5x + z = 115$
 $5(20) + z = 115$
 $100 + z = 115$
 $z = 15$

$y = 4(20)$
 $y = 80$

$(20, 80, 15)$

Ex 2:

- ① $x - y + 2z = 4$
- ② $x - 3z = 1$ step 1 → $x = 3z + 1$
- ③ $2y - z = -15$

① $(3z + 1) - y + 2z = 4$ ③ $2y - z = -15$

No x in this equation

step 2 $5z - y + 1 = 4$ Solved for z → $z = 2y + 15$

$5z - y = 3$

step 3 $5(2y + 15) - y = 3$ 2 new equations w/ same variables. Choose your method: Sub or Elim?

$10y + 75 - y = 3$ $z = 2\left(\frac{22}{9}\right) + 15$

$9y + 75 = 3$ $z = \frac{44}{9} + \frac{135}{9}$

$9y = -72$ $z = \frac{179}{9}$

$y = -8$

step 4 $x - 3\left(\frac{179}{9}\right) = 1$ $x = \frac{182}{3}$

$x - \frac{179}{3} = \frac{3}{3}$

$x = \frac{182}{3}$

Yay! Fraction answers!

$\left(\frac{182}{3}, \frac{22}{9}, \frac{179}{9}\right)$

Homework:

1.4 Part 1 p.34 #1,23-26

And

1.4 Part 2 below

To be a system solution, an ordered triple must satisfy ALL three equations

Decide whether the given ordered triple is a solution of the system.

1. $(0, 0, 3)$

$3x + 4y + z = 3$

$-2x + 7y + 2z = 6$

$-10x + 12y - z = -3$

2. $(-1, -2, 5)$

$2x + y - 5z = -29$

$6x + 4y - z = -19$

$x + y + 2z = 7$

3. $(0, 0, 0)$

$x + y + z = 0$

$2x + 3y - z = 0$

$3x + y - 4z = 1$

4. $(-1, -3, -2)$

$x - 5y + 6z = 2$

$3x - y + 8z = -16$

$4x + 2y - 7z = 4$

5. $(5, 7, 1)$

$x + y + z = 13$

$2x - 7y + 5z = -34$

$3x + y + 4z = 25$

6. $(-4, 8, -9)$

$x + 2y - 3z = 39$

$2x + y - 7z = -63$

$3x + y + z = -13$



1,2,4 are
3,5,6 are not