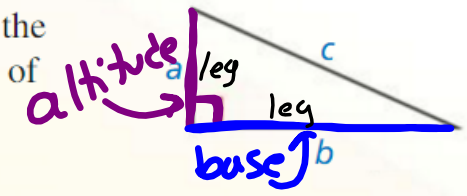
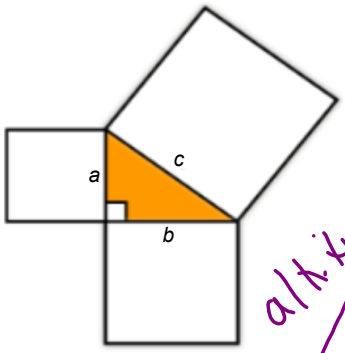
Your Name
Mrs. Theo
3/28/22
Notes

Pythagorean Theorems and Triples

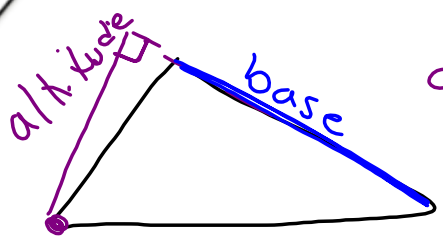
Theorem 9.1 Pythagorean Theorem

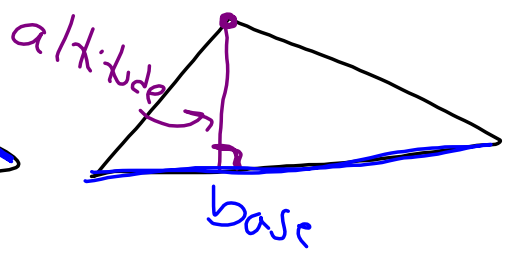
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.





If right triangle, then $a^2 + b^2 = c^2$.

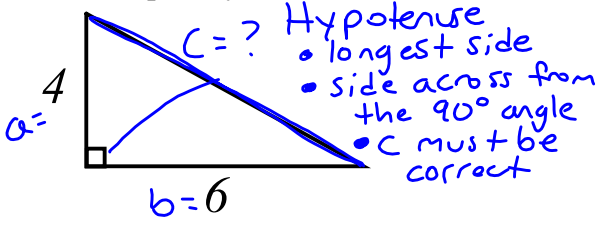




1- 9.1 - Pythagorean Theorem and Triples Notes

Example #1

Find the lengths of the missing sides of the right triangles below. Express your answer as a reduced radical.



Hypotenuse
- longest side
- side across from the 90° angle
- c must be correct

$a^2 + b^2 = c^2$

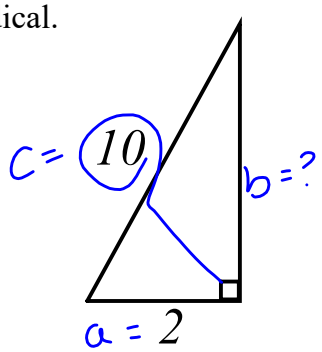
$4^2 + 6^2 = c^2$

$16 + 36 = c^2$

$\sqrt{52} = \sqrt{c^2}$

$7.211 = c$

$7.21 = c$



$a^2 + b^2 = c^2$

$2^2 + b^2 = 10^2$

$4 + b^2 = 100$

$-4 \quad -4$

$\sqrt{b^2} = \sqrt{96}$

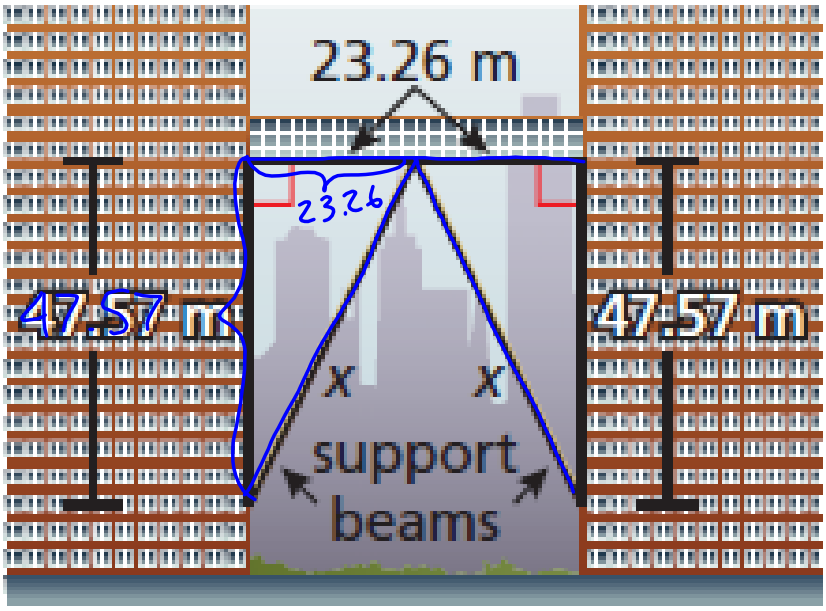
$b = 9.797$

$b = 9.80$

1- 9.1 - Pythagorean Theorem and Triples Notes

Example #2

The skyscrapers shown are connected by a skywalk with support beams. Use the Pythagorean Theorem to approximate the length of each support beam.



23.26

47.57 $c = ?$

$a^2 + 6^2 = c^2$

$23.26^2 + 47.57^2 = c^2$

$541.02 + 2262.9 = c^2$

$\sqrt{2803.92} = \sqrt{c^2}$

$c = 52.95$

Pythagorean Triples - All sides are positive integers

Big 4! (Most common)

a b c

3-4-5	5-12-13	8-15-17	7-24-25
$3^2 + 4^2 = 5^2$	$5^2 + 12^2 = 13^2$	$8^2 + 15^2 = 17^2$	$7^2 + 24^2 = 25^2$

3 4 5

12 5 13

Others

9-40-41

11-60-61

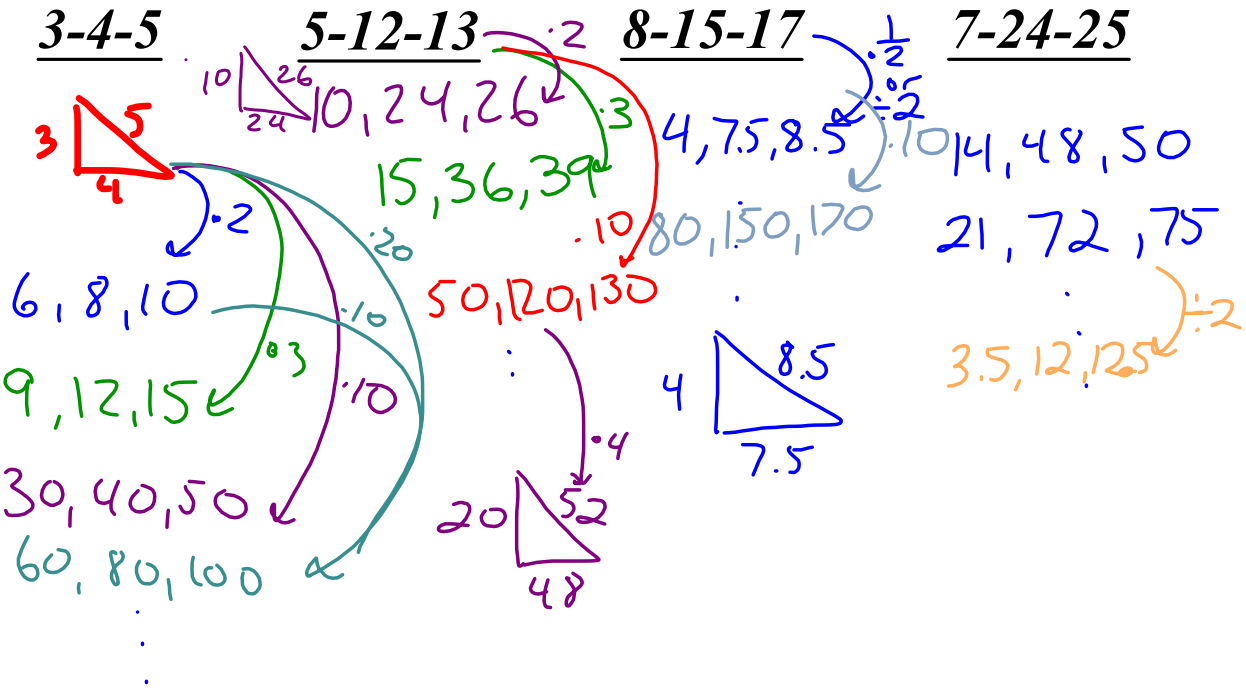
20-21-29

12-35-37

1- 9.1 - Pythagorean Theorem and Triples Notes

Families of Pythagorean Triples

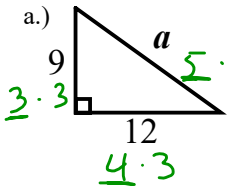
Multiples or divisors of common triples



Example #3

Find the missing side lengths in the right triangles below using pythagorean Triples.

a.)

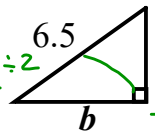


3·3 — 4·3 — 5·3 = 15

3-4-5

$a = 15$

b.)

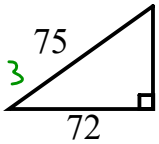


5-12-13

13 ÷ 2 — 5 ÷ 2 — 12 ÷ 2 = 6

$b = 6$

c.)

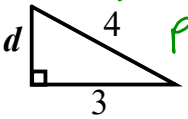


7-24-25

25·3 — 7·3 = 21 — 24·3

$c = 21$

d.)



Not a Pythagorean Triple

$d^2 + 3^2 = 4^2$

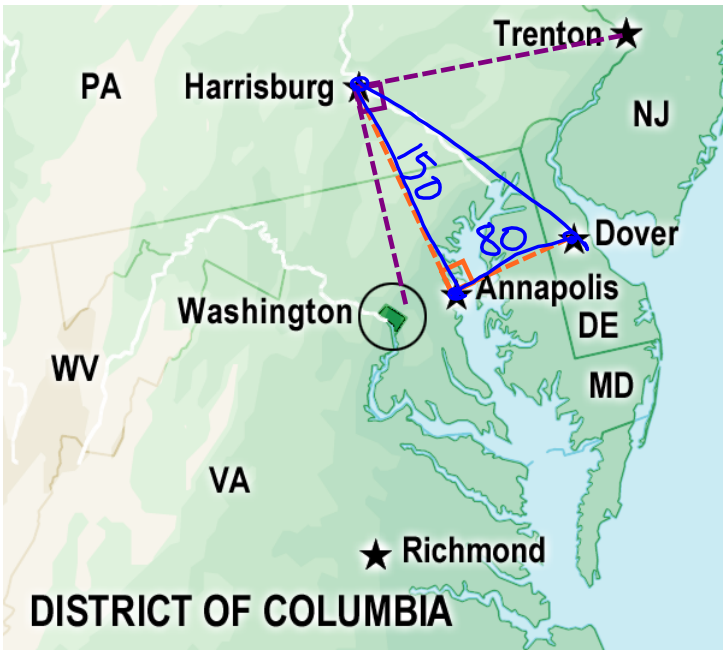
$d^2 + 9 = 16$

$d^2 = 7$

$d = \sqrt{7}$

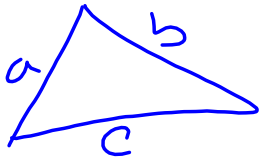
Example #4

It is about 80 mi. from Dover, DE to Annapolis, MD.
It is about 150 mi. from Annapolis, MD to Harrisburg, PA.
About how far is it from Harrisburg, PA to Dover, DE?



8 - 15 - 17 ·10
80 - 150 - 170
170 miles

Qualifications to be a triangle



$a + b > c$

Think about it...

If the sum of two sides is not greater than the third, it would not create a triangle at all!

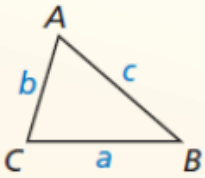
When a + b = c then it would only create a straight line!

1- 9.1 - Pythagorean Theorem and Triples Notes

Theorem 9.3 Pythagorean Inequalities Theorem

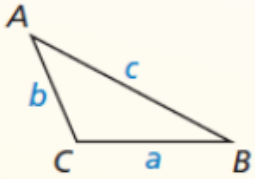
For any $\triangle ABC$, where c is the length of the longest side, the following statements are true.

If $a^2 + b^2 > c^2$, then $\triangle ABC$ is acute.



$a^2 + b^2 > c^2$

If $a^2 + b^2 < c^2$, then $\triangle ABC$ is obtuse.



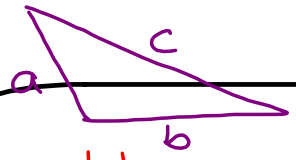
$a^2 + b^2 < c^2$

Converse of the Pythagorean Theorem

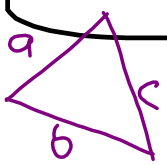
If $a^2 + b^2 = c^2$ ---→ Then it is right

Pythagorean Inequalities Theorems

If $a^2 + b^2 < c^2$ ---→ Then it is obtuse



If $a^2 + b^2 > c^2$ ---→ Then it is acute



Example #5

Are these triangles right, obtuse or acute?

3 - 5 - 7

longest is 'c'

$3+5>7$ ✓

$3^2+5^2 \lessgtr 7^2$

$9+25 \lessgtr 49$

$34 < 49$

34 is less than 49

So this is an obtuse Δ

8 - 9 - 5

longest

$8+5>9$ ✓

$8^2+5^2 \lessgtr 9^2$

$64+25 \quad 81$

$89 > 81$

89 is greater than 81

So this is an Acute triangle

6 - 3 - 2

longest

$2+3 \lessgtr 6$

not even a triangle!

$a+b \not> c$

$2 \quad 3 \quad 6$

Homework:

Classify the triangles below as acute, right, or obtuse.

a.) Side of 7, 9, and 11.

$7+9>11$ ✓

$7^2+9^2 \lessgtr 11^2$

$49+81>121$

Acute

b.) Sides of 6, 8, and 10.

$(3\text{-}4\text{-}5)\cdot 2$

$6+8>10$ ✓

$6^2+8^2 \lessgtr 10^2$

$36+64=100$

Right

c.) Side lengths of 4.3, 5.2 and 7.1 (all inches).

$4.3+5.2>7.1$ ✓

$4.3^2+5.2^2 \lessgtr 7.1^2$

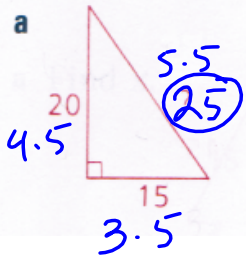
$45.53 < 50.41$

Obtuse

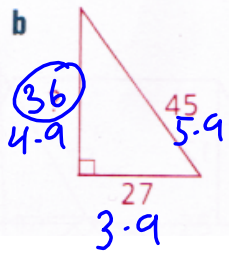
1- 9.1 - Pythagorean Theorem and Triples Notes

1 (3, 4, 5)

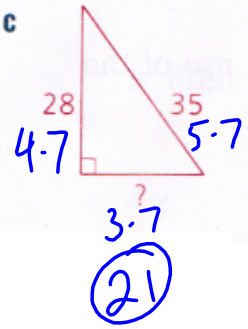
a



b

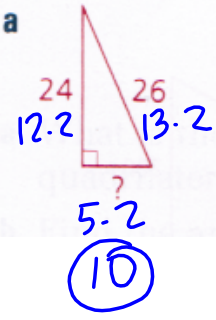


c



2 (5, 12, 13)

a



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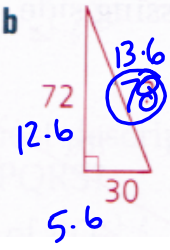
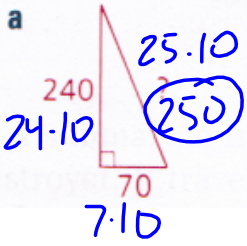


3 (7, 24, 25)

a



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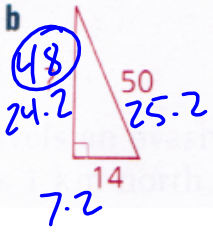
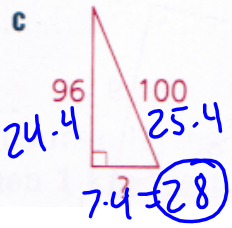
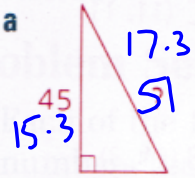


c



4 (8, 15, 17)

a



b